STATE OF ILLINOIS
WILLIAM G. STRATTON, *Governor*
DEPARTMENT OF REGISTRATION AND EDUCATION
VERA M. BINKS, *Director*

STUDIES OF WATERFLOOD PERFORMANCE

III. – Use of Network Models

Walter Rose

DIVISION OF THE
ILLINOIS STATE GEOLOGICAL SURVEY
JOHN C. FRYE, *Chief* URBANA

CIRCULAR 237 1957

STUDIES OF WATERFLOOD PERFORMANCE
III. — USE OF NETWORK MODELS

Walter Rose

ABSTRACT

This study was undertaken to further the utility of network model methods for solving petroleum engineering problems. Such models are used to analyze a mathematical equivalent of a representative volume element of a porous continuum, giving attention to the microscopic details of pore sizes, shapes, and degree of interconnections. From this analysis, macroscopic effects can be synthesized, such as the dependence of capillary pressure and relative permeability on fluid saturation and distribution; and, in principle, important characteristics of reservoir performance may be inferred.

Formerly only two-dimensional networks have been used because the computations for three-dimensional models are laborious. Moreover, analog model solutions have been restricted to steady-state flow problems. Now certain new computer methods make it possible to study the characteristics of larger and more complex three-dimensional networks. Continuous rather than step functions of pore size distribution can be employed with a variable pore shape and a randomized degree of interconnections in the selected networks. This minimizes arbitrary boundaries and achieves a model which in the statistical sense has a high degree of point-to-point identity with the prototype reservoir rock. In consequence, results are possible which may be supposed to have a direct bearing on the mechanics of petroleum recovery, such as waterflooding.

INTRODUCTION

According to one point of view, the success of waterflooding operations depends on conditions of favorable sweep efficiency and favorable displacement efficiency. Sweep efficiency is largely a function of the selected flood pattern as modified by gravity effects and the anisotropies of the formation itself. Displacement efficiency, of course, is zero in the unswept regions of the reservoir; but in the swept regions it will have a finite value controlled by a host of vaguely understood determining factors (Rose, 1957).

In field operations exact values for the sweep and displacement efficiencies are not known separately, and frequently their combined effect can be only guessed. Thus, in practice, the operator has a rough estimate of the amount of oil left in the reservoir after primary depletion, and years later he notes he has successfully produced a certain additional volume of oil. The ratio of the oil produced by the secondary recovery operation to that left after primary depletion is the recovery factor, which in the fundamental sense is directly related to the product of the sweep and displacement efficiencies.

Obviously it would be a great advantage to have a quantitative value for the recovery factor before beginning a secondary recovery operation. The economics of the project could then be predicted with reasonable certainty. As it turns out, much effort has been directed to this purpose (Muskat, 1949, Ch. 12, 14) through the development of core analysis, the study of laboratory models, and the use of various analytic methods of inquiry. Two categories of difficulties are encountered in any attempt to predict recovery factors, however. First, it is not practical to stipulate the exact character of the reservoir system with regard to initial and boundary conditions of fluid saturations and pressures, nor can the variations of permeability and porosity be specified. Second, the inherent non-steady nature of secondary recovery processes makes exact solutions largely intractable.

This paper discusses a new method of studying displacement efficiencies. The method involves the construction of a model of a small volume element of the reservoir so that the transient history of the movement of the flood front through the prototype reservoir can be synthesized. Such an application represents a microscopic approach, calculated to show the dependence of the displacement efficiency (in swept regions) on the various controlling factors (for example, injection rate, initial saturation conditions, and reservoir rock texture).

Emphasizing the displacement efficiency component of the recovery factor parameter (to the exclusion of the sweep efficiency component) appears justified, partly because more is known already about sweep efficiencies. They are generally high when the permeability stratification and mobility ratio are favorable, having values approaching unity (100 percent) in some cases (Aronofsky and Ramey, 1956). Sweep efficiencies, in addition, are not wholly dependent on the nature of the reservoir system, but can be controlled and improved by operating practices such as the proper geometric location of input and production wells, selective plugging (or fracturing and acidizing) certain zones, infill drilling, control of mobility ratio, making use of gravity effects (Jordan et al., 1957) and so forth.

Factors that affect displacement efficiency are far less understood, as will be seen. It is true that if one were able to imagine the proper initial saturation conditions, a simple restored state type of core analysis experiment would give the displacement efficiency directly. The literature contains, however, many contradictory results obtained in this way, especially as bearing on the relationship between displacement efficiency and flood rate (Calhoun and LaRue, 1951; Moore and Slobod, 1956). Considering the complications of laboratory procedure, the inconclusiveness of reports on core analysis is not surprising.

The method of study here described involves construction of so-called network models of small volume elements of the reservoir prototype. Recent work by Fatt (1956) has suggested this application possibility. The network model, as used by Fatt, was intended actually to represent the microscopic texture of reservoir rock, and he attempted to construct therefrom the detailed distribution of the immiscible fluid saturants of the pore spaces which would result from fluid-fluid displacement processes. Fatt also attempted to establish the relationships between relative permeabilities which were associated with the interstitial fluid distributions.

Muskat (1949) had previously cited two reasons why it is necessary "to visualize the problem of fluid flow through porous media from a statistical (macroscopic) point of view." He mentioned: "from a practical point of view it is not feasible to give quantitative description of the microscopic geometry of the flow system, as defined by myriads of complicated passageways bounded by the ensemble of grains and their cementing material." Then he noted: "... previous studies of the classical hydrodynamics equations show that they are far too complex to permit integration for any such systems of irregular and multiply-connected flow channels, comprising the pores of oil-bearing rocks, even if they could be quantitatively defined."

In effect, Fatt challenged the necessary correctness of Muskat's first objection, and he circumvented the need to consider the truth of the second.

In the exact sense, Muskat is right in saying that the detailed microscopic character of porous media, such as reservoir rock, cannot be quantitatively defined, much less used to solve the fundamental hydrodynamic equations of fluid flow and displacement. Following Fatt, however, this paper asserts that limiting cases of rock texture can be stipulated with sufficient exactness so that network models can be constructed which will be useful in establishing (in a semi-quantitative way) the separate importance of the various factors that control fluid flow and fluid displacement (that is, displacement efficiencies and recovery factors).

A network model may be defined as the mathematical representation of a system of interconnected pore spaces in which there is an element of point-to-point identity with the reservoir prototype as regards pore shapes, lengths, widths, and orientation. To be perfectly general, the character of the reservoir rock prototype might be stipulated in accordance with the proposition that upon selecting a random point located in the volume element of interest, there will be statistical probabilities that: 1) the point is located in pore space rather than in space occupied by the solid phase; 2) wherever the point is located, it is a certain distance from a pore-solid phase boundary; 3) said interfacial boundary has a certain shape, orientation, and surface roughness character; and 4) said pore space is located a certain distance from an interpore junction point and filled either with interstitial water, oil, or free gas. Hence, the same statistical probabilities should be the character of the network model if exact similitude is to be achieved.

Although Fatt implied the theoretical possibility of constructing exact network models, he did not impose such severe requirements on his initial work. In point of fact, he found it convenient (and perhaps thought it adequate to the intended purpose) to consider only pore size distribution and degree of pore interconnection, and he made the latter regular and uniform in a way which he himself noted was both arbitrary and unrealistic. Thus, the network models employed in Fatt's pioneering work did not take into account pore shapes, variable degree of interconnection, and other textural characteristics of real reservoir rock.

Fatt's work was also limited in other ways because he did not have access to the high-speed computer facilities which the present paper shows are required for any comprehensive use of network models. (For example, Fatt says in response to the discussion of Rose et al. (Fatt, 1956): "... the use of high speed

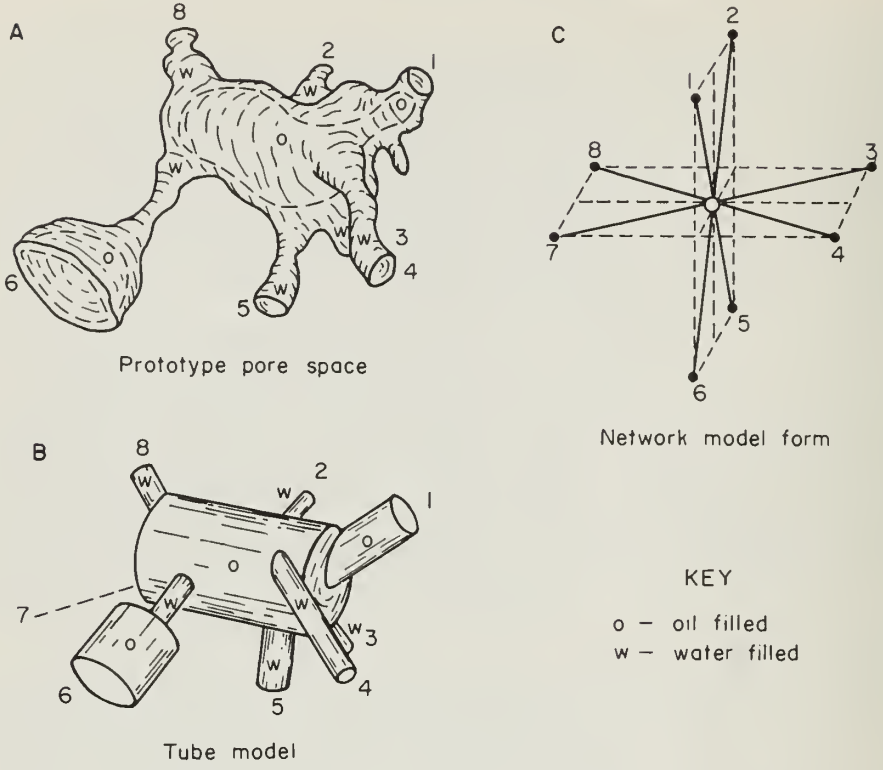

Fig. 1. - Achieving similitude between reservoir pore space prototype and tube and network model forms.

computers ... will make possible tests of the network model that I could not even consider while limited to my pencil-and-paper accounting procedure.")

Fatt, therefore, considered rather small networks (both with regard to number of pores and number of pore sizes) in only two dimensions, and restricted his work to static equilibrium saturation end-points and steady-state, single-phase flow; moreover, laboriously constructed analog models were required to investigate relative permeability and resistivity index phenomena. Finally, initial and boundary conditions, and implied histories of displacement, were frequently arbitrary and unrealistic in deference to the need for simplicity.

In the present paper the view is still held that exact network model analogies are at least theoretically possible; and, moreover, as high-speed computer facilities are now available, the usefulness and validity of network model studies can be increased greatly. Thus, three-dimensional models composed of tens of thousands of pores and hundreds of pore sizes can be imagined where the transient effects leading to the static equilibrium and steady state end-points can be considered; moreover the need to construct analog models of the network model may be obviated largely by certain newly developed computing methods as described herein.

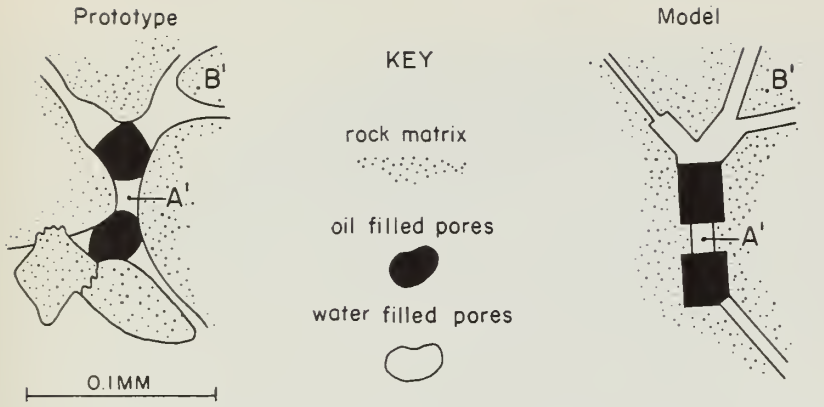

Fig. 2. - Model equivalent of random section through a porous rock as seen in thin section (after Bruce and Welge, 1947). Points "A" and "B" are corresponding positions in model and prototype.

Figure 1 illustrates similitude between prototype rock and tube and network models.

Figure 2 also shows, in two dimensions, the degree of correspondence between reservoir rock prototype and network model which is believed to be satisfactory for many application purposes. Points A and B are random locations in a random plane which passes through the reservoir of its network model. If there is point-to-point identity between an infinity of these points located in an infinity of plane sections in both model and prototype, then exact similitude has been achieved.

In order explicitly to consider flow and displacement phenomena, it is useful to choose simple pore shapes for the model (such as cylindrical tubes); therefore in practical work it is often necessary to make some departure from exact representation. This appears to introduce error of second order importance, however, if Carman's remarks (1937) about the relative unimportance of shapes of pore cross-sections can be accepted. Also, Rose and Cleary (1957) have established the near-equivalence of a pore doublet network (constructed from cylindrical tubes) to a sphere packing, assuming the latter is represented adequately by a network of interconnected pores having the shape of centrally-tapered hyperboles of revolution. This demonstrates further the relative unimportance of pore shapes.

PRINCIPLES OF NETWORK MODEL CONSTRUCTION

Figure 3 shows a network model form believed to be adequate for many purposes. It is a three-dimensional array of mesh points located at the corners of cubes (or tetrahedra), where each point is connected to adjacent points along the cube edges and certain specified face diagonals. These interconnections represent the relative symbolic position of pores, and the mesh points represent the relative positions of pore interconnections; but, as will be seen, exact locations are not indicated until pore length and diameters are specified. Since pore sizes can range from zero to some arbitrary maximum, geometry shows

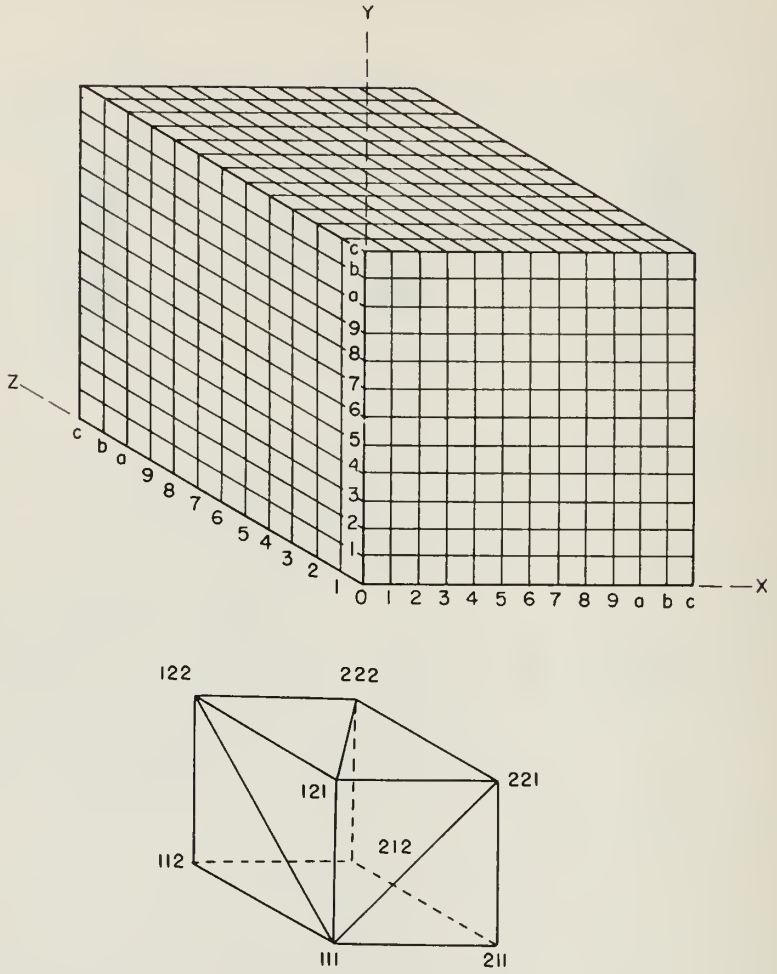

Fig. 3. - The tetrahedron (3-D) network form. Insert shows interconnection between eight adjacent mesh points.

that the degree of interconnection of pores (at mesh points) can range from zero to a maximum of 22 (that is, when all the pores joining at a given mesh point have a finite size).

In order to use the tetrahedral network model for solving problems, it must be possible to scale into the model values for the following textural characteristics of the reservoir rock prototype, namely: 1) porosity; 2) specific surface area; 3) tortuosity; 4) pore diameter sizes and size distribution; 5) type and degree of pore interconnection; and 6) pore lengths. (As implied above, pore shapes and surface roughness characteristics are not here considered. Obviously, it is also impossible to consider fluid-solid phase interactions, such as clay swelling effects, at this stage.)

Fatt, in his previous work, ignored items 1), 2), 3) and part of 5). He specified item 4) by a pore-size distribution curve, item 5) he referred to degree of interpore connections by his "beta" factor, and item 6) he referred to various arbitrary functions.

The Pore Interconnection Concept

Regarding the interconnection between pores, it can be shown that merely specifying the beta factor is not definitive. For example, the two-dimensional triple hexagonal pattern and the three-dimensional cubic pattern both have a beta factor of 10 (that is, each pore is connected to five others at each terminal end). However, in tracing possible paths between two adjacent mesh points, both patterns have one one-pore path and four three-pore paths; but the triple hexagonal form has two two-pore paths and the cubic form has no two-pore paths. Interconnecting paths of four or more continuous pores perhaps can be neglected. (This is certainly implied by the analytics of the various pore-doublet studies which have been made [Rose and Witherspoon, 1956; and Rose and Cleary, 1957].) Nevertheless, geometry will show considerable differences in this respect between the two subject network patterns.

Fatt stated that relative permeability phenomena were more dependent on the beta factor than on the pore size distribution; hence, it may be predicted that the nature of the interpore connections (in addition to the number of interpore connections) will have a similar importance. Therefore, it is convenient to introduce the notation shown in table 1 to more completely specify network form, where the three or four numbers following the beta factor indicate respectively the single, double, triple, and quadruple pore path interconnections between adjacent mesh point junctions.

Table 1. - Certain Regular Network Model Forms

Name	Beta factor	Path characterizing number	A	B
Simple bundle	∞	1	-	-
Tetrahedral (3-D)	22	1 4 8	12	12
Cubic (3-D)	10	1 0 4 0	6	6
Rhombohedral (3-D)	10	1 0 3 0	8	4
Triple hexagonal (2-D)	10	1 2 4 (2, 4, 4)	6	6
Double hexagonal (2-D)	7	1 0 2 or 1 2 0	6	3
Square (2-D)	6	1 0 2	4	4
Single hexagonal (2-D)	4	1 0 0	3	3
Composite bundle	2	1	2	2

At this point it is worth while to note that Fatt's triple hexagonal pattern reduces to the double or single hexagonal patterns, or to the square pattern, by systematically designating certain pores as having zero size. Likewise, the three-dimensional tetrahedron pattern can be made to reduce to cubic and other three-dimensional forms (of lesser beta factor); and, in fact, if only one plane is considered, the various two-dimensional patterns discussed by Fatt also are obtained.

With such variations in mind, the possibility of randomly "knocking" out pores by specifying zero sizes can be considered. Thus, Fatt and his reviewers noted the relevancy of providing for random pore interconnections in network models. This was suggested by examination of thin sections, and presumably the zero-pore device mentioned above is a satisfactory way to achieve this.

(It may be assumed that the random location of zero-sized pores is a proper procedure only when dealing with large networks such as discussed in this paper. The general rule seems to hold that the network model must be large enough to have a character of homogeneity and isotropy, even though the microscopic elements, such as pore size and length, vary markedly from point to point.)

For such random interconnections, the beta factor and the characterizing number showing path interconnections will have values less than those indicated in table 1; moreover, such values will not necessarily be unique numbers descriptive of the situation at a particular set of adjacent mesh points, but rather they will be of statistical significance only (in a way related to the probability of zero-sized pores occurring).

A further comparison can be made, however, to emphasize the importance of considering the nature of interpore connection as well as degree of pore branching. Rose (1957) discussed the network model form of the (regular) rhombohedral packing of uniform-sized spheres. This form may be designated as "rhombohedron (3-D)," and it has each octrahedral pore junction connected to eight tetrahedral pore junctions (Rose, 1957a, fig. 1). Each of the eight tetrahedral pore junctions is connected to three other octrahedral pore junctions. In consequence, the beta factor is 10, as it is in the cubic (3-D) network form; but the path characterizing number of the cubic-network is (1040), whereas for the rhombohedron it is (1030). Thus it is seen that a single three-pore path is the difference between the simple and symetrical cubic form and the more complex and assymetrical rhombohedron form.

The Tetrahedron Model

The special advantage of the tetrahedron (3-D) network form (fig. 3 and table 1) is that it is characterized by a high degree of interpore connections and pore path possibilities; and in all probability, no reservoir rock prototype ever will be found in nature where such a high degree of pore branching will be required in the corresponding network model. Hence, assigning zero sizes to certain pores of the tetrahedron network will reduce the beta factor and alter the path characterizing number, in accordance with the probability of occurrence and location of the zero pores to be simulated. Explicitly, the beta factor for the case of no zero pores is given by:

$$\beta = A + B - 2 \qquad (1)$$

where A and B are the terms given in table 1 which indicate, respectively, the number of pores joining at any two adjacent mesh points. (Note - Evidently, $[\frac{A + B}{4}]$ defines approximately the ratio between total pores and total mesh points in a given network.)

It follows, then, that if the probability that any random pore in the network has zero size is P_z, the beta factor is reduced to:

$$\beta_z = [(1 - P_z)(\beta + 2)] - 2 \qquad (2)$$

Thus, referring again to table 1, it is seen that making one-third of the pores in the triple-hexagonal network of zero size, reduces the beta factor to six. Selecting these zero pores in a certain regular fashion results in producing a

square network form that has a path characterizing number of (102...); conversely, a random location of the zero pores causes the randomization of the beta factor and the path characterizing number throughout the resultant network. Likewise, the cubic network form can be derived from the tetrahedron (3-D) form by letting all of the diagonal pores be of zero size (that is, P_z = 0.5).

It is thus apparent that starting with the tetrahedron (3-D) network model form, many variations in model representation of prototype texture can be achieved by the simple device of assigning zero sizes for regularly or randomly selected pores, so that the resultant networks have any beta factor (less than 22) and any path characterizing number (less than 148...), as may be desired to simulate the pore texture of some given reservoir rock. The beta factor and the path characterizing number, moreover, can be made to vary around some mean value throughout the network as occasionally will better represent the anisotropies of reservoir rock, and this is achieved by random, rather than regular, location of the zero pores.

This paper does not contain detailed instructions for the construction of network models which will represent any particular reservoir prototype; it rather demonstrates that the construction possibility exists, and that it is now practical to think about building network models for solving problems. It is not essential to the present discussion, therefore, to prove that beta factors greater than 22 and pore characterizing numbers greater than (148...) never occur in nature, as already suggested by the thin sections shown by Winsauer et al. (1952).

It is also beyond the scope of this paper to actually construct in network form the model of any particular porous media system. For example, the network model form of a rhombohedral packing of uniform size spheres has already been discussed, and it is clearly implicit in the foregoing remarks that a modified tetrahedral network model form can be devised to have close similitude to porous media packings composed of randomly sized and shaped particles in random array. The latter, cemented or uncemented, and with or without other anisotropies, is equivalent to the natural sedimentary sandstones which comprise many petroleum reservoirs.

Having arrived at a suitable form for the network model (namely, the zero-pore modified tetrahedral form), there is still the problem of introducing control of porosity, permeability, pore size and length distribution, pore shape and tortuosity, specific surface area, and other intensive properties of texture, if a valid model of the reservoir prototype is to be achieved. There also is the problem of representing initial and boundary conditions, so that reservoir performance problems can be solved with the network model. These questions are answered in part in the succeeding sections of this paper. One final matter should be discussed before considering such applications, however.

Looking, for example, at the various two-dimensional network forms proposed by Fatt, one gains the feeling that permeability always will be directional, and that the regularity of pore orientation will be a limiting factor. This view, however, is believed to be unjustified, as suggested by the following considerations:

1) Given a network model which is big enough, the fluid flow capacity will not depend on direction any more than in the reservoir rock.

2) In any case, and at least from the microscopic viewpoint, flow direction is less important than pore size and pore path-interconnection. Actually, the angle of pore orientation implied by the network form is largely of symbolic significance.

3) Finally, the network model is obtained by superimposing a grid over the volume element of the reservoir rock of interest, and then scaling into the model the characteristics which are found at each mesh point. In accord with all approximation processes of this type, one always has the opportunity to choose smaller incremental values (a finer grid) in order to make the resultant step-functions more closely approximate the continuous functions of nature.

NETWORK MODEL CONSTRUCTION

Given the tetrahedron (fig. 3) as the network model form to be considered, the extensive and intensive properties of the system can be specified by introducing the following notation:

- r_i - radius of ith pore having ℓ_j length
- ℓ_j - length of jth pore having r_i radius
- V_{ij} - volume ($\pi r_{ij}^2 \ell_{ij}$) of r_i, ℓ_j pores
- A_{ij} - area ($2\pi r_{ij} \ell_{ij}$) of r_i, ℓ_j pores
- N_{ij} - number of r_i, ℓ_j pores
- N - total number of all pore elements in network
- N_L - number of pore following any X, Y, Z path (i.e., a measure of the external dimensions of the model)
- V_B - bulk volume
- V_P - pore volume
- f - fractional porosity
- S - fractional saturation
- A - specific surface area per unit pore volume
- A_I - interfacial specific surface area per unit pore volume
- C - length scaling factor, cms. per arbitrary length unit
- w - subscript, designating wetting phase saturation
- V_B/L - model cross-sectional area perpendicular to flow direction
- L - flow length of model

Accordingly, the bulk volume, cross-sectional area, and length of the network model, respectively, are given by:

$$V_B = [\bar{\ell} N_L C]^3 \; ; \; \frac{V_B}{L} = [\bar{\ell} N_L C]^2 \; ; \; L = [\bar{\ell} N_L C] \qquad (3)$$

where the average pore length, $\bar{\ell}$, is simply: $\bar{\ell} = \dfrac{\Sigma \ell_j N_{ij}}{N}$

Pore volume is given by: $V_P = C^3 \Sigma N_{ij} V_{ij}$; so porosity can be expressed as:

$$f = \frac{\Sigma N_{ij} V_{ij}}{[\ell N_L]^3} \tag{4}$$

Likewise, the saturation terms will have the form:

$$S_w = \frac{\Sigma N_{iw} V_{ijw}}{\Sigma N_{ij} V_{ij}} \tag{5}$$

and the specific surface area terms will have the form:

$$A = \frac{\Sigma N_{ij} a_{ij}}{C \Sigma N_{ij} V_{ij}} \tag{6}$$

$$A_I = \frac{\pi}{C} [\Sigma \beta_{iw} N_{iw} r_{iw}^2 / \Sigma N_{ij} V_{ij} \tag{7}$$

where β_{iw} is the local beta factor for each wetting-phase saturated pore r_{iw} connected to nonwetting-phase saturated pores.

If pore size and length distribution functions are given, Equations (3), (4) and (6) can be solved explicitly, upon specifying the magnitude of the scaling factor, C. Likewise, Equations (5) and (7) are solvable if the distribution of wetting and nonwetting phases is known.

The Permeability Problem

To calculate permeability, the need for high-speed computing methods is apparent. Fatt (1956) has shown that the application of Kirkhoff's laws leads to the consequence that an (N + 1) order determinant must be solved in order to obtain the specific resistance of a network made up of N elements. For the problem at hand, where a tetrahedral form network of 1,728 tubes (12^3) is being considered, this introduces severe limitations if "brute force" methods are employed to solve the requisite set of simultaneous equations.

For example, the ILLIAC digital computer facilities at the University of Illinois is limited to solving 143 simultaneous equations by the direct method; but recently (Golub, 1957) a new program has been written which makes it possible to solve problems involving, for example, 12^3 elements with a reasonable amount of labor. In principle, then, it is simply necessary to specify the pore radius and length distribution functions, together with the impermeable-barrier boundary conditions, and an explicit solution is obtained.

Although it is beyond the scope of this paper to consider the details of such computing possibilities, it can be noted that foreknowledge of the eigenvalues of the matrix (specifically, the ratio of the minimum to the maximum values) greatly increases the rate of convergence and stability of the solution. Also to be noted is the limitation of Golub's program to the solution of simultaneous equations where zero coefficients are prevalent, as this diminishes the computer storage problem. Fortunately, the latter condition is met in the problem at hand.

When more than one immiscible fluid is saturating the pore space, the reduced conductivities to fluid flow for each fluid (that is, the relative per-

meabilities) also can be calculated by the method described above. This is done by modifying the network through locating zero pores except where the fluid of interest is to be found, resulting in a variable decrease in the effective beta factor and path characterizing number associated with each mesh point. In the limit, these numbers approach zero, which is to say that the relative permeability goes to zero as path continuity is broken, even though a finite (and perhaps large) amount of the fluid of interest still saturates the pore space.

The Tortuosity Problem

The foregoing focuses attention on the important tortuosity concept about which there has been so much confusion. This may be defined as the constant of proportionality which appears in the Kozeny-Carman equation for fluid flow as:

$$K = f/ T t_s A^2 \qquad (8)$$

where K is permeability, f is porosity, t_s is the pore shape factor (having a constant value usually close to 2), A is the specific surface area per unit pore volume, and T is the tortuosity. Figure 4 is a nomograph which is useful for rapid calculations with Equation (8).

In the derivation of the Kozeny-Carman equation, T takes on the dimensional sense of being the square of the ratio of the actual pore path length to the bed thickness; and in the case of unconsolidated sandstones, this pore orientation factor apparently has a real and simple geometric meaning. But, as Carman notes (1956), some workers consider it unreasonable to retain this concept in the case of consolidated porous media because of the high values of tortuosity which have been measured (for example, Rose and Bruce, 1949, report values as high as 400 for certain well-cemented sands).

Nevertheless, the view taken here is that the tortuosity term has a real physical significance, even though it may be difficult to arrive at a general definition of it for use in describing consolidated porous media. This can be illustrated nicely by reference to the network model, for here tortuous paths easily are visualized. Thus, it is clear that pore orientation (network form) and pore interconnection (the beta factor and the path characterizing number) have a direct influence on the flow conductivity, as does the frequency and distribution of zero pores. Finally, pore size distribution itself also will be important.

Thus, given the square network form, it is apparent that the value for tortuosity will approximate unity if all the pores are of the same size and length, but under these same conditions it will be greater than unity for the single-hexagonal network form (namely, $T \cong 2.25$). As the distribution of pore sizes is broadened, however, tortuosity should increase for all network forms when these various pore sizes are randomly distributed in space. This is because it is likely that the most conductive paths through the randomly located larger pores will tend not to be the shortest-distance paths. Likewise, decreasing the beta factor and/or the path characterization number, as for example by increasing the number of zero pores and/or decreasing the saturation of the pore saturant of interest, will also cause the tortuosity to be increased.

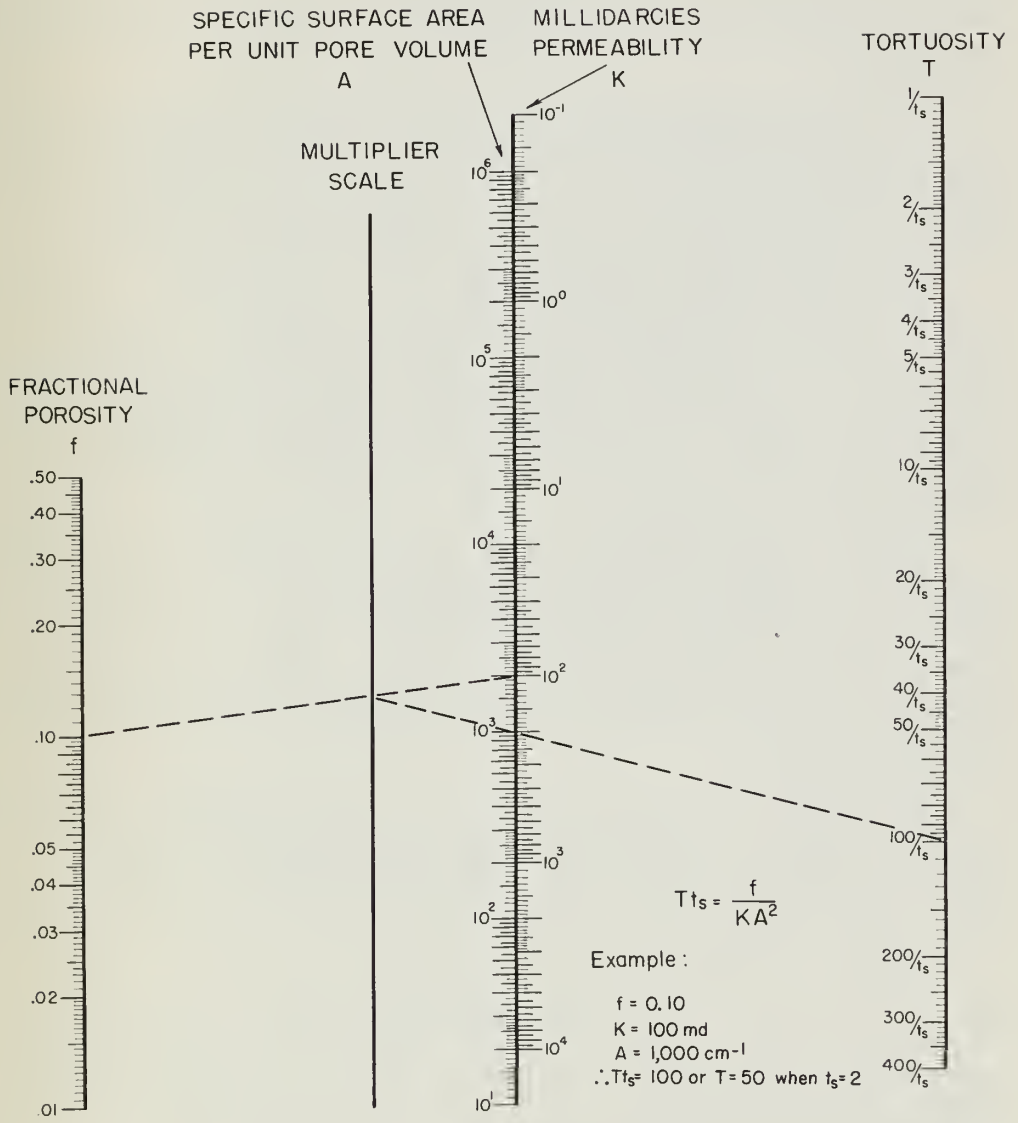

Fig. 4. - The Kozeny Equation nomograph.

It is in these senses that it is believed valid to speak of the tortuosity concept, both with reference to network models and the reservoir rock prototypes they represent. Thus, pore orientation, interconnection, size-distribution, and saturation conditions (and perhaps even mobility ratio), all determine the tortuous path which will be followed during the viscous flow of a particular pore saturant.

Indeed, if a physical picture is desired, tortuosity can be thought of in terms of the ratio of the average "maximum velocity streamline length" to the bed thickness. "Maximum velocity streamline length" is a heretofore undefined parameter which is believed to have important significance, and is defined in terms of the microscopic velocities along streamline paths, as observed during viscous flow through porous media.

Elementary considerations show that streamline paths never cross, and that the velocities are vectors which vary from maxima in the central pore spaces to minima which are zero at solid phase (pore wall) boundaries, or which are finite at fluid-fluid boundaries (and at fluid-solid boundaries if the flowing fluid is a gas). The microscopic velocities also may vary along the length of each streamline path, due to the involutions and convolutions of the pore wall boundaries, but this latter velocity variation is not relevant to the present discussion.

Figure 5 shows in schematic detail a random section taken more or less perpendicular to the direction of flow through a porous medium, where isovelocity contours have been drawn. A collection of contiguous sections, therefore, will define the loci of the interwoven threads which are the "maximum velocity streamline paths," such as P_1, P_2 and P_3. Apparently, the average distance of all of these maximum velocity paths can be sensibly greater than the geometric tortuous path, and certainly greater than the bed length, as depicted also in figure 5.

Thus, given a volume element of a porous medium of interest, it is clear that a finite number of maximum velocity streamline paths enter, and that an equal number leave; moreover, considering again the single-hexagonal network form comprised of pores of different conductivity, the streamline paths will be larger than the geometric paths, which in turn are larger than the bed length. This is shown schematically by figure 6.

Moreover, it should be noted that tortuosity will be further increased by introducing zero pores (for example, by "knocking" some of the pores out of the network, or filling them with some immiscible and non-flowing fluid), and by changing the network form as regards the beta factor and/or the path characterizing number. Thus, the paradox discussed by Carman (1956, p. 50) appears to be resolved, and his statement that "... the term, tortuosity factor, is (no) longer justifiable ..." appears to be contradicted.

Although the above remarks give only a physical picture of tortuosity without an analytic formulation, the latter, of course, is still desired, and will be the subject of further investigation in this laboratory.

The problem at hand, however, is how to construct network models of particular reservoir systems so that fluid distribution and fluid flow problems can be solved. To do so, all of the textural properties of the reservoir rock prototype must be scaled into the model, including tortuosity as well as

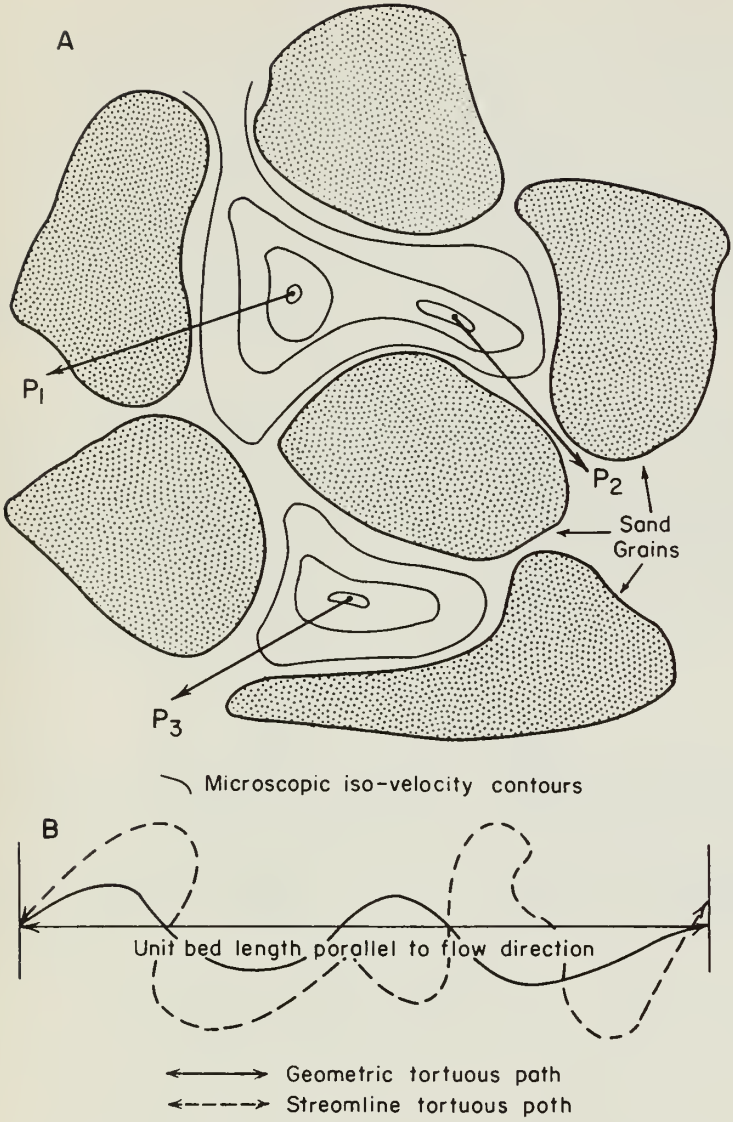

Fig. 5. - A schematic representation of the iso-velocity contours, and the maximum velocity streamline loci (P_1, P_2 and P_3), depicted in microscopic section perpendicular to the direction of flow. Lower drawing shows difference between geometric and streamline paths.

porosity, permeability, capillary pressure-saturation relation, saturation and saturation distribution, specific surface area, beta factor, pore path interconnections, etc. In particular cases, values for these parameters will be known from core analysis, as they characterize particular reservoir rock prototypes. For example, tortuosity is obtained by formation factor (electrical) measurements, as first proposed by Wyllie and Rose (1950), specific surface area is obtained by the method of Rose and Wyllie (1950), and the other values are obtained by standard methods.

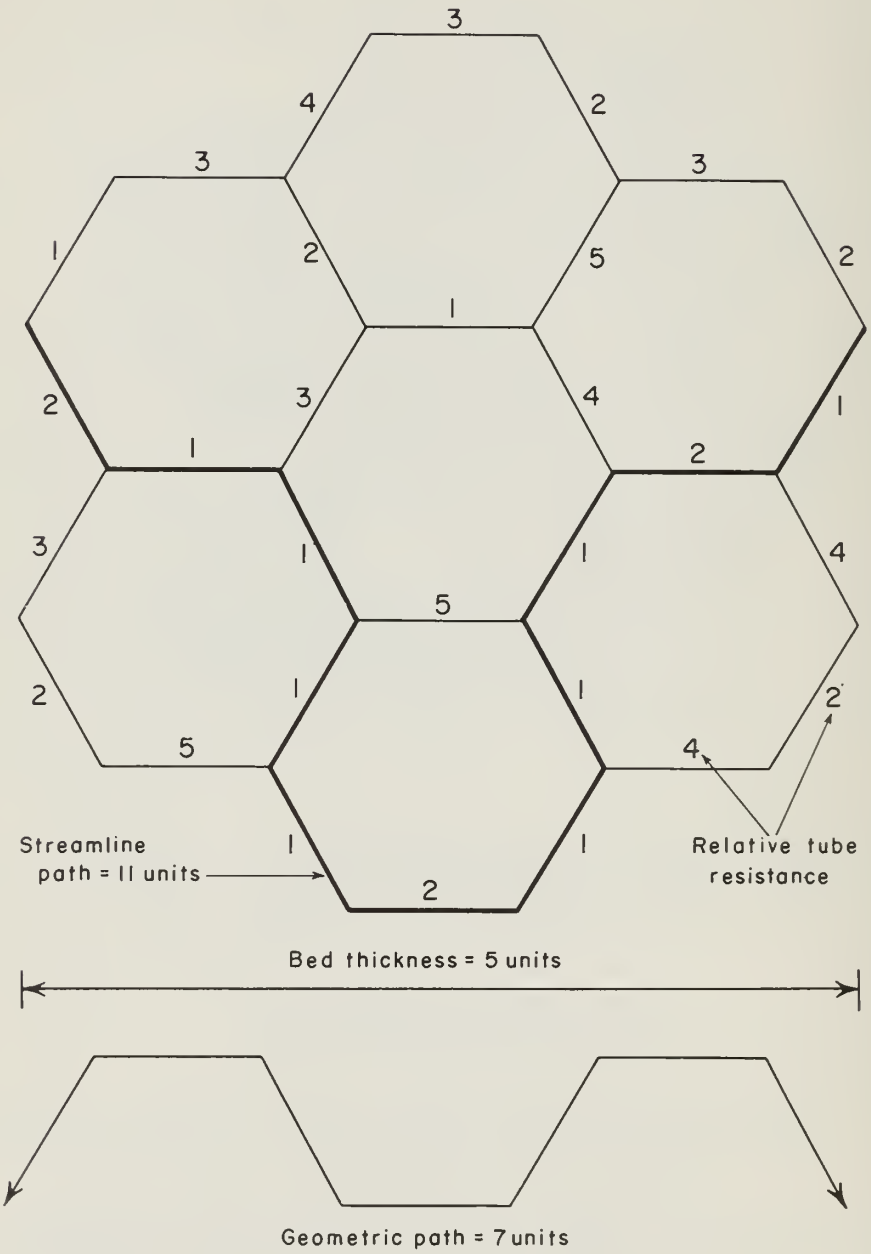

Fig. 6. - Tortuosity and geometric tortuosity illustrated by the single-hexagonal network form oriented parallel to the direction of flow. Numbers beside tube elements refer to relative resistance to flow.

Equation (3) through (7), and the succeeding discussion of ILLIAC programming of the permeability problem, suggest ways to build network models scaled with all textural properties except tortuosity which, at present, must be scaled by indirect methods. For example, a network form with suitable beta factor and path characterization number may be chosen to give a reasonable representation of the "geometric" tortuosity of the prototype. The actual tortuosity of the resultant network model can then be calculated from Equation (8), and if this is sensibly less than the value characteristic of the prototype, modifications are then introduced to achieve similitude. For example, a different pore size and length distribution can be selected so that gross features such as porosity and permeability are not altered, and apparently this is a valid and necessary procedure if a more reasonable scaling of tortuosity is to be obtained.

Perhaps better procedures can be proposed in time, but it should be emphasized that the tortuosity concept cannot be neglected in the construction of valid network models. The unsupported statement of Gates (in his written discussion appended to Fatt's paper, 1956) must be rejected, where he says: "The use of the (network) model eliminates the 'bugger factor,' sometimes termed tortuosity, which is used to make theoretical calculations on the bundle of tube models fit experimental data." Clearly, the network model which fails to consider the tortuous texture of reservoir rock will not give reasonable solutions of reservoir problems, except perhaps in trivial cases.

NETWORK MODEL USE

The intent of this paper is to show that valid network models of porous reservoir rock can be constructed and used in the solution of reservoir performance problems, such as those encountered in the study of waterflooding. To this end, Fatt's pioneering work has been taken as the point of departure, and it has been shown that bigger (3-D) and better-scaled models can be constructed. In the following description of their use, all solutions refer to digital computer methods.

Fatt's previous proposal to solve flow problems by building RC-electrical analogs of the network models, of course, is impractical when considering large networks, and when desiring numerical accuracy in the results. Analog models are also unnecessary, as will be shown.

The first step is to choose a network model form (for example, the tetrahedral 3-D form) and size, and then to scale in the prototype textural properties as discussed in the previous section. For example, given the 12^3 model of figure 3, and the pore size distribution of figure 7, and choosing Fatt's functional relationship between pore radius and length as:

$$\ell_i = C_1 r_i^a \tag{9}$$

porosity values can be calculated as shown in table 2.

In a like manner, given the network form and the pore size and length distribution functions, and given the sequence of the capillary pressure desaturation process as obtained by Fatt's method, a variety of problems can be investigated by the simple device of variously regarding the arbitrary pore size

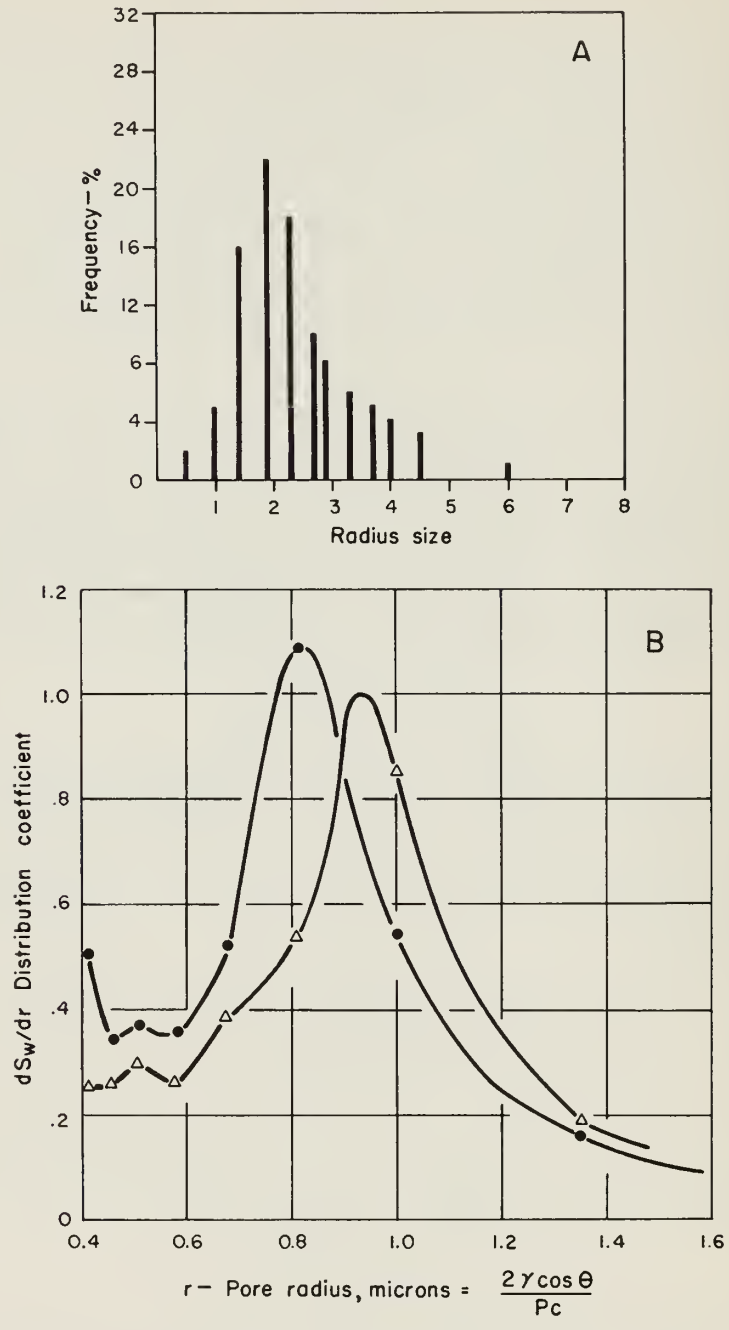

Fig. 7. - A network model pore-size distribution function, as suggested by the replot of the data of Rose and Bruce (1949).

Note: $\int_0^r (dS_w/dr)\,(dr) = S_w$.

Table 2. - Various Porosity Values for the Cubic Network Form

f	a	C_1
0.477	-2	10^3
0.242	-1	10^3
0.117	0	10^3
0.028	1	10^3
0.005	2	10^3
0.279	1	10^2
0.050	2	10^2
0.500	2	10

numbers and their relationship to actual pore size. Thus, taking the pore size distribution of figure 7 again, and the desaturation sequence given as table I-III of Fatt's paper (1956), the representations of figure 8 are obtained, which show how skewing the significance of the size distribution function alters the capillary pressure curve. Previously herein (table 1), it was implied that the length scaling factor, C, should have a constant value, but in obtaining figure 8 an arbitrary connection was assumed. Specifically, the relationships shown in table 3 were selected.

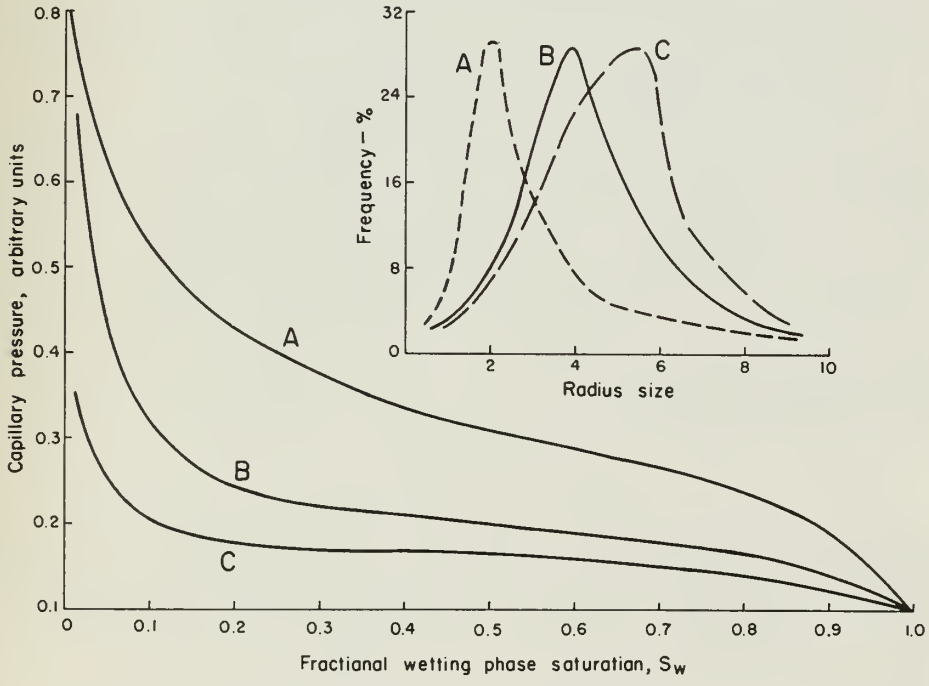

Fig. 8. - Relationship between capillary pressure curves and pore size distribution as determined by network model analysis (table 3).

Table 3. - Arbitrary Designations of Pore Size

Saturation Stage	Fatt's tube radius designation	Equivalent value for Curve "A"	Equivalent value for Curve "B"	Equivalent value for Curve "C"
(a)	4.96	10.0	10.0	10.0
(b)	4.45	8.0	9.0	9.5
(c)	3.96	6.5	8.0	9.0
(d)	3.67	5.0	7.0	8.5
(e)	3.54	4.5	6.5	8.0
(f)	3.27	4.0	6.0	7.5
(g)	2.92	3.5	5.5	7.0
(h)	2.62	3.0	5.0	6.5
(i)	2.32	2.5	4.5	6.0
(j)	1.91	2.0	3.5	5.5
(k)	1.53	1.5	2.5	3.5
(l)	1.00	1.0	1.5	2.0
(m)	0.54	0.5	0.5	0.5

The above discussion and table point to one of several features of flexibility by which the results of one programmed problem become applicable to the solution of a number of specific network problems. This, of course, is a common characteristic of all model procedures, and it is now emphasized in connection with network model work because every valid simplification must be utilized if practical solutions are to be obtained.

Varying the magnitude of the scaling parameters is an obvious device, but less apparent is the idea that the initial specification of the pore size distribution function only freezes the relative size of the various pores (that is, which pore is bigger) without saying anything about their relative magnitudes (that is, how much bigger). In fact, it is this latter device which has been used in the construction of table 3. It follows as a general statement, therefore, that any selected network will provide solutions applicable to a limitless number of assumed textural properties of the prototype. The variable properties include permeability, relative permeability, specific surface area, tortuosity, etc., in addition to porosity and the capillary pressure relationship which have been explicitly treated above.

Programming Procedures

The next step in problem solving, after having chosen the network form and size, and having scaled in the desired textural parameters, is to program the problem for digital computing. Programming is done, for example, by locating the information of table 4 in the computer's memory and/or input source of instructions.

Table 4. - Requirements for Digital Computer Programming

1) Mesh point and pore element initial and boundary information (N.B. One address in the computer memory stores this information for each element):
   - pore size (zero or finite)
   - pore shape (cylindrical or other)
   - pore length (random, constant, or related to pore size)
   - initial pore saturant (oil, water, or gas)
   - identification of adjacent pores (related to beta factor).

2) Gross boundary conditions:
   - source and sink location
   - impermeable flow barriers
   - other relevant streamline boundaries
   - boundary pressure conditions in the various fluid phases.

3) Intended scanning sequence to simulate the desired time-history of fluid movement and/or saturation change:
   - the restored state program
   - the primary production process program
   - the projected secondary recovery process program.

4) Desired print-out sequence:
   - initial condition information about the magnitude of the prototype's various (intensive) textural characteristics in the various senses which are to be considered
   - solution of the conductivity (permeability) problem
   - relaxation information to give the relative flow capacities (see below)
   - specification of the saturation changes
   - depiction of the flood-front advance as a function of time.

5) Intermediate storage program which tags relevant information about altered conditions at the memory address of each element and mesh point at the end of each scan, such that subsequent scans provide print-out data which move forward in time.

The computer solution method implied by the organization of table 4 involves storing necessary information in the memory, and necessary instructions either in the memory or as a part of the input tape program, so that a sequence of events occur which is equivalent to whatever is supposed to be happening (or whatever is assumed will happen) in the prototype system. The print-out, then, is the final result. Preferably this should be organized in a form convenient for immediate analysis and interpretation. Any experienced programmer can organize problems of the complexity contemplated in this paper.

Of special interest is the scanning feature, which is a device for systematically, sequentially and repetiously "asking" the following type questions with reference to each pore element:

1) What is the pore size and length and what is the pore saturant?
2) What are the size, lengths, and pore saturants of the contiguous pore elements and what are their spatial location?
3) If there were no fluid flow (static equilibrium), would the pressure be sufficient to cause a change in the nature of the pore saturant in response to capillary pressure effects?
4) If condition 3) holds, is there a connected entry path for the displacing phase, and a connected exit path for the displaced phase?
5) For whatever boundary pressure conditions are assumed, what would be the local mesh point pressure (cf. relaxation method discussion, below) and what in consequence would be the relative fluid conductivity?

It is to be noted that all of these questions can be phrased in such a way that a "yes, no" answer suffices, which explains how the unique scanning procedure is so easily adapted to digital computing. For example, the question about pore size is asked with reference to a size interval, so that the answer can be yes or no. Similarly, when the question is asked of a given pore element, can a displacing phase enter and can a displaced phase escape, the answer will follow a pattern such as the following:

> Yes, if the pressure conditions are right, and if there is a continuous entry path back to the displacing phase source at the network boundary, and if there is a continuous escape path for the displaced phase connected to the external sink boundary; otherwise, no.

Finally, when the question is asked about the local mesh point pressure, it is done in accordance with the principle of the relaxation method, where a "no" answer says an assumed pressure value does not satisfy the equation of continuity and the imposed boundary conditions. In this case, a "no" answer will be repeated until the correct assumption is made, and then the "yes" answer tags the pressure approximation so that the information is available for later use.

The above type questions are asked until the answers no longer change, and then a print-out is made which will give the new conditions of saturation, saturation change, and saturation distribution. At this point, the input program invokes new pressure conditions, and the scanning process is repeated until the desired complete history of fluid movement and fluid displacement has been observed, recorded, and interpreted.

Concurrent with these scanning steps, of course, the effective permeability relationship to saturation and saturation distribution are obtained for each pore saturant, by the method of solving the network matrix already discussed. A more interesting possibility is to make use of established relaxation methods (already available in ILLIAC's library of programs) to trace the history of flood-front advance and to evaluate relative permeabilities.

An Application of the Relaxation Method

Dykstra and Parsons (1952) have discussed the application of relaxation methods to solve certain oilfield model problems. In their work, they thought of porous media in the macroscopic sense as a continuous conductor, which (as an approximation) could be represented by a regular grid of interconnected tubes. These tubes would have the same radius and length if an isotropic medium is being considered, so that upon applying the equation of continuity at each grid intersection (mesh point), a correct guess about the local pressures throughout the network is indicated by:

$$\lim \Sigma \Delta P = 0 \qquad (10)$$

where the ΔP's are the pressure differences between the local mesh point of interest and all other contiguous mesh points (one tube element distance away). The local P values themselves initially are guessed, making use of whatever physical intuition is possible, so that the first set of summations have finite values greater or less than zero. By successive approximation, then, the pressure values are adjusted upwards or downwards (using the initial residual to improve the indications of physical intuition) until all the summations approach zero closely. (A general form for the residuals, δ, to be relaxed to zero is: $-\frac{\beta}{2} P_j + \sum^{\beta/2} P_i$ where P_i refers to the pressure at the mesh point of interest, and the P_j's are the pressures at the adjacent mesh points.) Finally, a decision is made that sufficient accuracy has been obtained in the result, and that further relaxations would only entail unprofitable continued computation. A balance also can be made between the grid spacing and the number of intended computations, so that the maximum accuracy is obtained with the least amount of effort.

In network model analysis, analogous relaxation procedures can be developed, and these are especially adaptable to digital computing. In this case, there is no need to superimpose a relaxation grid over the model, since the latter already is in network form. That is, the flow in and out of given mesh points can be considered directly on the basis of conservation of matter, so that local pressure points can be evaluated. The relaxation approximations are carried forth, however, using the relation:

$$\lim \Sigma \frac{r_i^4}{\ell_j} \Delta P_{ij} = 0 \qquad (11)$$

where Equation (11) reduces to Equation (10) in the event all the pore sizes and lengths are the same. Note, the fluid conductance of a network made up of tubes in series and parallel is:

$$\sum^{n} \frac{1}{\sum^{m} \ell_i / r_i^4}$$

Figures 9 and 10 depict the solution of relative permeability problems by the relaxation method. In figure 9, an array of uniform-sized tube elements is assumed, having the interconnection given by the state "f" desaturation process of Fatt's table I-III (1956); and noted thereon are the relaxed pressure

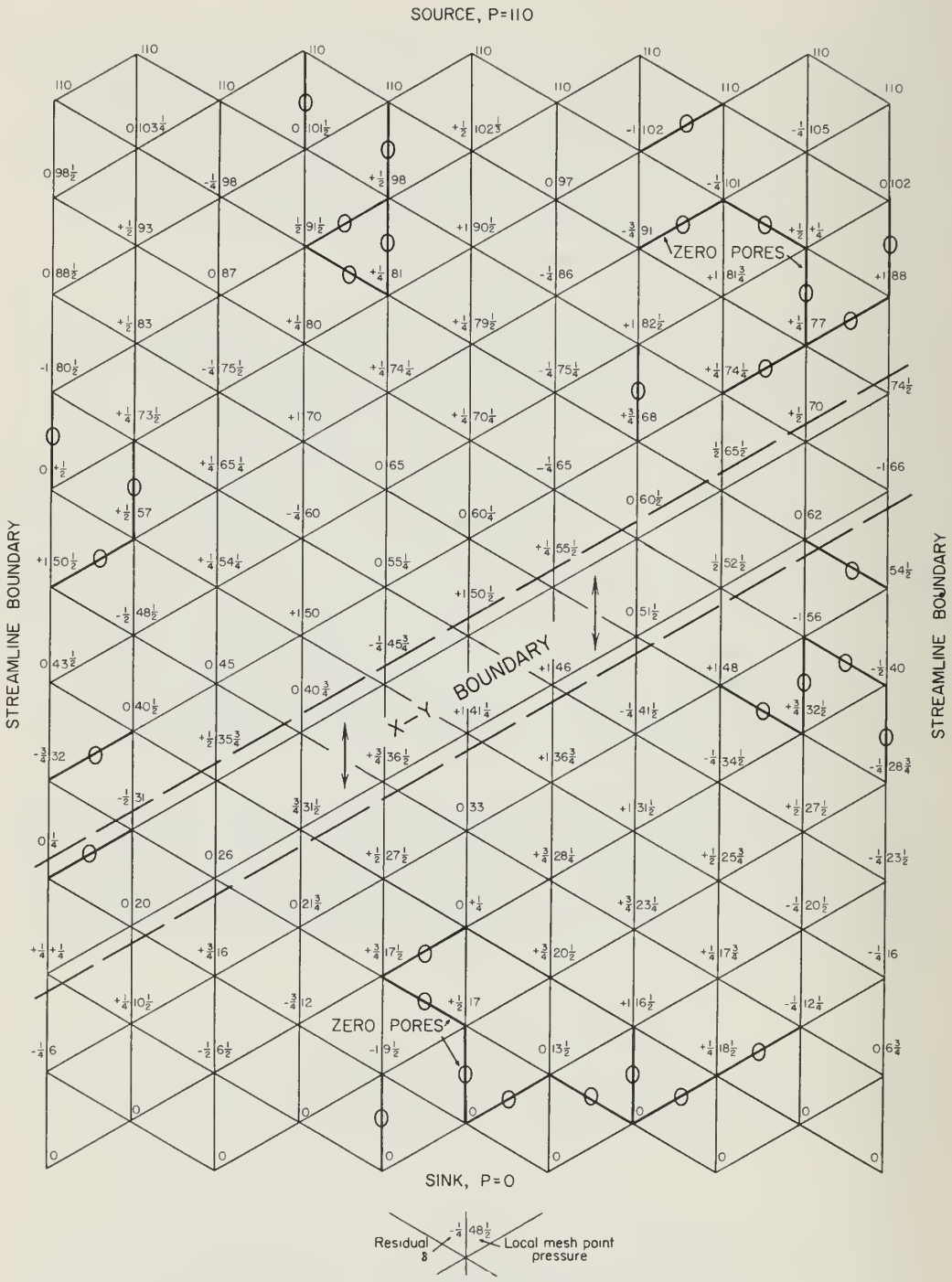

Fig. 9. - Triple-hexagonal network model showing mesh point pressures in the wetting phase saturated portion, as obtained by relaxation analysis.

values (and residuals) locally occurring at each mesh point. Relative permeability of the wetting phase can then be calculated by relating the flow across any arbitrary boundary at saturation, S_w, to the flow when S_w is unity, as given by:

$$K_{rw} = U/V \tag{12}$$

where U is calculated by the summation of Equation (11), considering only the wetting phase saturated pores going across the arbitrary boundary; and V is calculated by the summation of Equation (11), considering all the pores going across this boundary and the ΔP's when S_w is unity. For example, the wetting phase saturated pore elements in figure 9, crossing the XY boundary, can be selected to calculate U, and these pores plus the zero-pores (oil saturated) which go across the XY boundary can be selected to calculate V.

Figure 10 is another problem of relative permeability of the wetting phase for the same triple-hexagonal pattern of uniform-sized pores, interconnected according to the "h" desaturation stage of Fatt's table I-III. In this case, a field map has been drawn by using the relaxed pressure values as guides to draw arbitrary equipotentials and streamlines. This device provides an alternate method of graphical solution, and table 5 shows some results which have been obtained.

Table 5. - Typical Relative Permeability Values

S_w	K_{rw}	Notes
0.90	0.840	Calculated by Equation (12)
0.70	0.548	Calculated by Equation (12)
0.70	0.554	Calculated by field mapping technique of Moore (1952)

Reference to figure 1-II of Fatt's paper (1956) shows that equivalent results have been obtained. Thus established is the general validity of the analysis methods which give the data of table 5, through the demonstrated conformance to the more laboriously obtained analog model methods employed by Fatt. This conformance is of added interest, for it will be recalled that Fatt assumed a random desaturation sequence and that table 5 shows data pertaining to a capillary pressure desaturation sequence. This seems to imply that, given a random location of pores of random sizes, there is little difference between the consequence of the two mechanisms. If fully established, this circumstance could be utilized to great advantage in simplifying the programming of mixture-flow problems because pore size need not be considered explicitly in developing the sequence of saturation change.

Obviously, relative permeabilities of the nonwetting phase also can be calculated by the methods given above, where the first (that is, Equation 12) is suitable for digital computing, and the second (that is, the graphical method) is of interest in pointing to the valid representation of a discontinuous network by a continuous medium. More important is the application of the latter to visualize the transient flood-front advance as fluid saturations are changing with time and position throughout the network during fluid displacement.

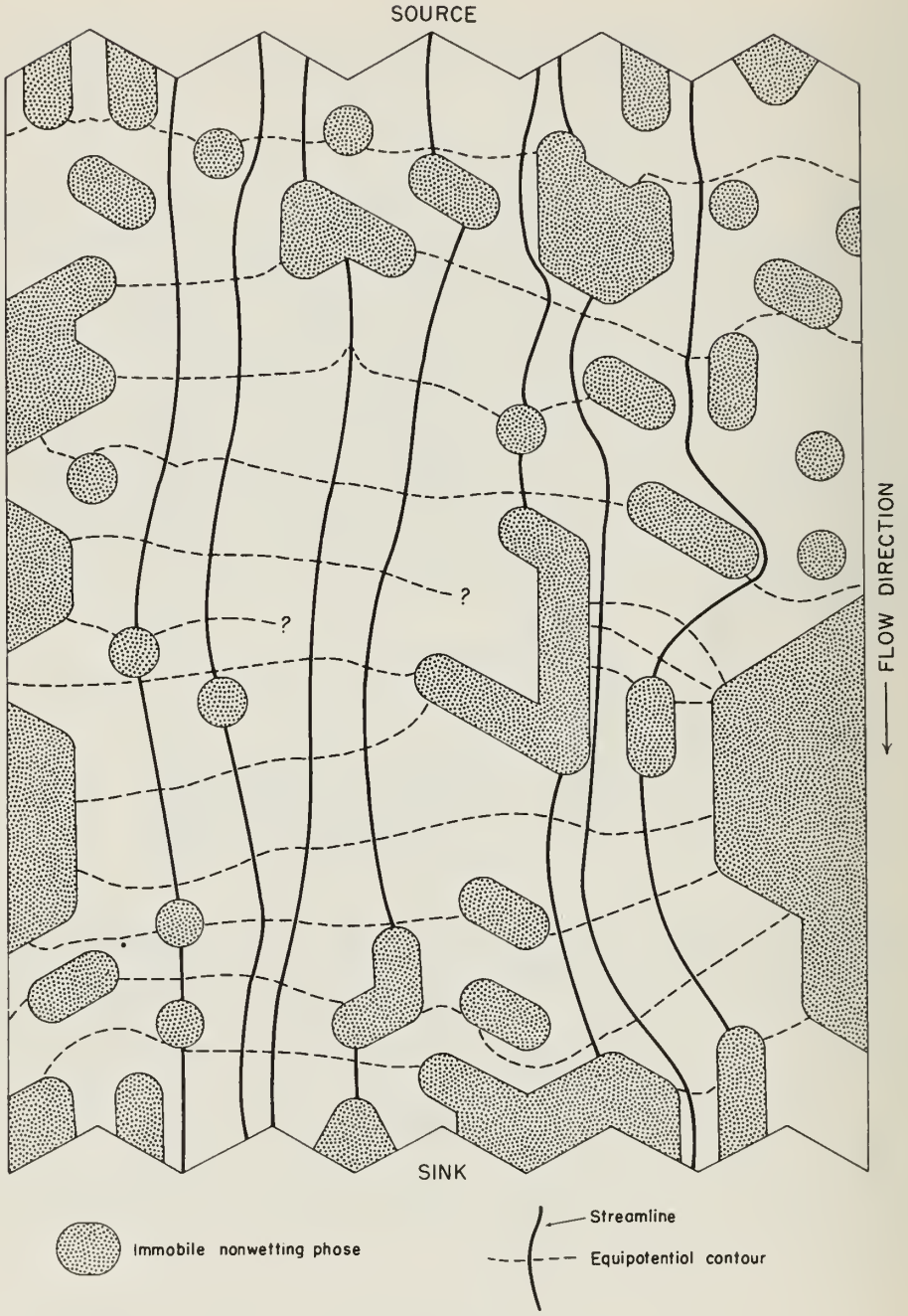

Fig. 10. - Triple-hexagonal network model with superimposed field map of streamlines and equipotential contours through the wetting phase saturated portion. Equipotential contours obtained from relaxation analysis; flow tubes, by the method of Moore (1952).

Other Applications

It will be recalled that Fatt originally used certain simplifications for tracing saturation changes involving the following:

1) There is a continuous path for the entry of a displacing phase into the particular pore element of interest; and
2) The static equilibrium boundary pressures in the displacing and displaced phase reservoirs creates a capillary pressure condition that means the displacing phase should enter, and the displaced phase should leave, the particular pore element of interest.

These simplified conditions inadequately represent what actually happens. First, there must be an exit path for the displaced phase as Rose et al. asserted (cf. discussion following Fatt's paper, 1956). Second, it is the internal and local pressures, and not the external boundary pressures, which determine whether or not imbibition or drainage displacement will occur. And as noted above, the relevant mesh point pressure values can be obtained by a programmed relaxation process. This means, of course, that the transient flood-front advance can be obtained as a succession of steady-states, according to standard methods (cf. for example, Muskat, 1949).

The foregoing suggests an application possibility of extreme importance, since heretofore it has been practical to consider only the gross aspects of flood-front advance, such as those related to gross boundary conditions, and even then most studies have been limited to a consideration of isotropic reservoirs and unity mobility ratio. The network model, however, provides a way to synthesize the complete history of displacement processes, starting with the microscopic details. Nonuniform reservoir anisotropies, complex initial and boundary conditions, and non-unity mobility ratio situations can be handled as simply as more trivial cases.

Still another application possibility is to program network analyzer solutions in two successive stages. The first part can be a conventional analysis wherein the network model has microscopic point-to-point similarity with a small volume element of the reservoir of interest. After the details of the displacement process have been evaluated, a second network model is constructed to represent the gross aspects of the reservoir at large. In such an application, each tube group in the second network has a length and radius such that it represents the porosity and permeability of the volume elements which are the prototype of the first network model.

For example, let L_x, L_y and L_z be the external dimensions of the volume element, and choose the second network of cubic form, and let r_x, r_y and r_z be the radii of the three tubes in the coordinate directions which together are to represent the volume element. Then, the permeability in the coordinate directions, k_x, k_y and k_z, and the porosity will be given by:

$$k_x = f\, r_x^2 / 8 \qquad k_y = f\, r_y^2 / 8 \qquad k_z = f\, r_z^2 / 8$$

$$f = \frac{r_x^2 L_x + r_y^2 L_y + r_z^2 L_z}{L_x L_y L_z} \tag{13}$$

Hence, by Equation (13), one can calculate the length and radius of the tube elements of the second network, which represent the permeability and porosity of the volume element prototype of the first network model. This approach would seem to permit analysis of cross-flow phenomena, stratification effects, and productivity indices of complex reservoir systems.

The above remarks about the importance of information on non-steady-state flow do not signify that there will be no future interest in relative permeability values for steady-state flow. In fact Richardson (1957) recently has shown the great utility of steady-state data for waterflood calculations. It is unfortunate, but a fact, that very little relative permeability data have been obtained to date and reported in the literature, a scarcity due for the most part to the difficulty of laboratory procedure. It is of interest, therefore, that analysis can be made of three-phase relative-permeability relationships using network model methods.

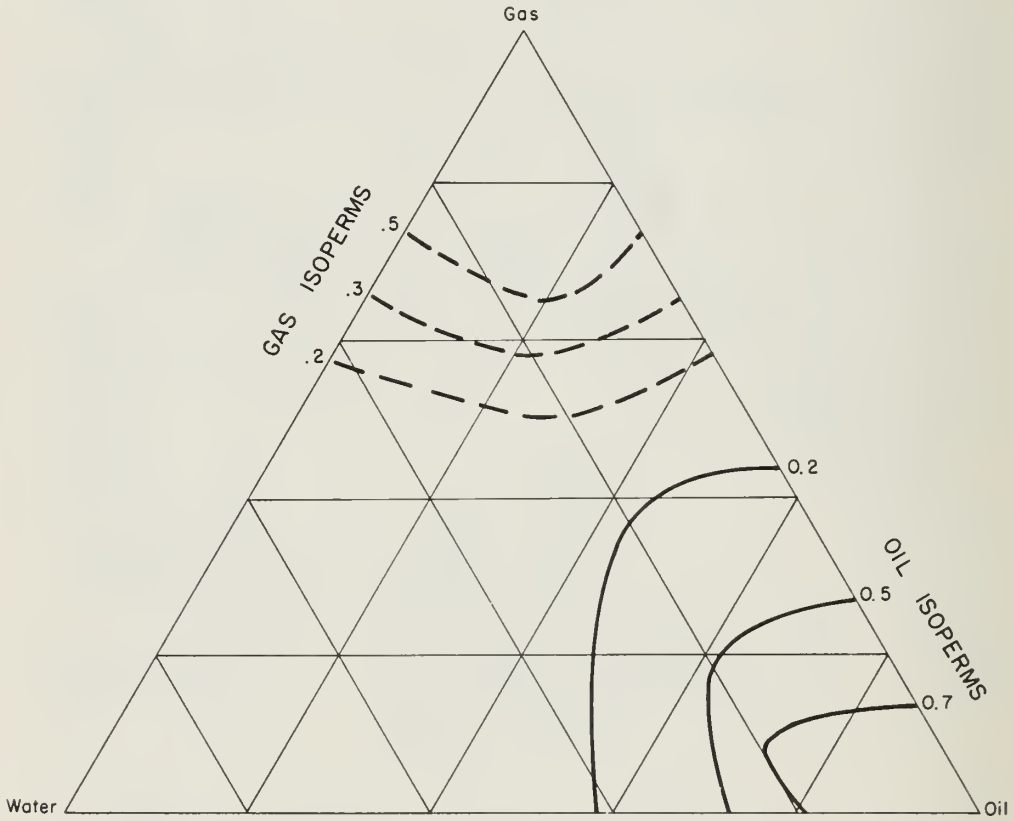

Fig. 11. - Ternary relative permeability diagram obtained from network model analysis.

For example, figure 11 is a conventional ternary plot of three-phase relative-permeability data obtained using a network system having the pore sizes of figure 7, and the methods of study discussed above and previously (Rose 1957). As can be seen, the water and oil isoperms are similar to those

reported previously (Muskat, 1949), but the gas isoperms suggest a higher conductivity for the gas in the three-phase saturation region than in the two-phase region. This is not an unreasonable expectation in accordance with the idea that the previously established paths of oil continuity in the three-phase region will aid the flow of gas. Hence, figure 11 may be taken as reasonably representative of relative permeability data, and the network model methods of analysis reported in this paper may be taken as valid approaches to an understanding of the performance of petroleum reservoirs.

CONCLUSIONS

Model studies are necessary in order to understand the details of petroleum recovery because field observation methods are inadequate and analytic studies are difficult. The network model offers especially unusual possibilities for evaluating and predicting future reservoir performance. For example, they provide a direct means for determining the displacement efficiency which indicates the success of waterflooding operations. Thus, this paper has been written with the general objective of furthering the utility of network model methods for solving reservoir engineering problems. Specific accomplishments include:

1) A statement of the importance of the displacement efficiency concept, and the unique ways this may be evaluated by network model analysis.

2) An enumeration of the reservoir rock intensive and extensive properties which must be scaled in the network model to achieve similitude, and an indication of how this scaling may be done.

3) The development of the "path characterization number" concept, the idea of "zero" pores, and notions regarding "maximum velocity streamlines." These innovations have led to new views about the important tortuosity concept which minimize the importance of "geometric" tortuosities.

4) An application of probability theory which asserts that a random point selected in prototype space has a certain probability of being located in a pore, and that this probability should characterize the model at all points. Other probabilities refer to defining the size, shape, orientation, and continuity of pores; to the pore saturants; to the proximity of fluid-fluid boundaries to solid phase surfaces; and to the size, shape, orientation, and continuity of these interfaces.

5) The procedure for describing the transients of flood-front advance so that factors affecting recovery can be evaluated through an application of the relaxation method.

6) The proof of the necessity for considering large (3-D) networks, and hence an indication of the necessity for digital computing methods.

7) An analysis of the requirements for programming various categories of network model problems on digital computers such as ILLIAC.

8) The indication that single network model solutions will have general application to a variety of problems, through control of scaling factors and other arbitrary specifications relating model to prototype.

9) The results of certain preliminary network model studies, establishing the validity and utility of the methods outlined in this paper.

Much remains to be done. Building and using network models require a heretofore unknown accuracy in specification of prototype characteristics. The development of digital computing methods is only in its infancy. Numerical checks, not yet available, are necessary to prove the full value of the various application possibilities. Still and all, the considerations treated in this paper add strength to the view that network model methods of analysis offer the clue, not elsewhere available, for finding practical solutions to important reservoir problems.

Acknowledgment

Helpful discussions with Paul Witherspoon and David Geiger during the early stages of this work must be acknowledged. Don Pierre provided some of the calculations, as did Richard Strauser of the Department of Mining and Metallurgical Engineering, University of Illinois.

REFERENCES

Aronofsky, J. S., and Ramey, H. J., Jr., 1956, Mobility-ratio - its influence on injection or production histories in five-spot waterflood: Trans. A.I.M.E., v. 207, p. 205.

Bruce, W. A., and Welge, H. J., 1948, The restored-state method for determination of oil in place and connate water: Drilling and Production Practice, 1947, p. 166, A.P.I., New York.

Calhoun, John, and LaRue, J. W., 1951, The effect of velocity in waterflooding: Producers Monthly, v. 15, no. 6, p. 21.

Carman, P. C., 1937, Fluid flow through granular beds: Trans. Inst. Chem. Engineers, v. 15, p. 154.

Carman, P. C., 1956, Flow of gases through porous media: Academic Press Inc.

Dykstra, H., and Parsons, R. L., 1951, Relaxation methods applied to oil field research: Trans. A.I.M.E., v. 192, p. 227 .

Fatt, Irving, 1956, The network model of porous media. I. Capillary pressure characteristics. II. Dynamic properties of a single size tube network. III. Dynamic properties of networks with tube radius distribution: Jour. Pet. Tech., v. 8, no. 7, p. 144, July, 1956. (Also discussions appended thereto by Gates, Josendal, and Rose et al., p. 177, 178, 179; also Trans. A.I.M.E., v. 207, p. 144.)

Golub, Gene, 1957, University of Illinois ILLIAC Library program of routines.

Jordan, J. K., McCardell, W. M., and Hocott, C. R., 1957, Effect of rate on oil recovery by waterflooding: Oil and Gas Jour., v. 55, no. 19, p. 98.

Moore, A. D., 1952, Mapping techniques applied to fluid mapper patterns: Proc. A.I.E.E., v. 71, p. 1.

Moore, T. F., and Slobod, R. L., 1956, The effect of viscosity and capillarity on the displacement of oil by water: Producers Monthly, v. 20, no. 10, p. 20.

Muskat, Morris, 1949, Physical principles of oil production: McGraw Hill Book Co., New York.

Richardson, J. G., 1957, The calculation of waterflood recovery from steady-state relative permeability data: Jour. Pet. Technology, v. 9, no. 5, p. 64.

Rose, Walter, 1957, Studies of waterflood performance. I. Causes and character of residual oil: Illinois Geol. Survey Bull. 80, p. 147-163.

Rose, Walter, and Bruce, W. A., 1949, Evaluation of capillary character in petroleum reservoir rock: Trans. A.I.M.E., v. 186, p. 127.

Rose, Walter, and Cleary, James, Further indications of pore doublet theory: Producers Monthly, in press, August, 1956 (Illinois Geol. Survey Reprint Series)

Rose, Walter, and Witherspoon, Paul, 1956, Studies of waterflood performance. II. Trapping oil in a pore doublet: Illinois Geol. Survey Circ. 224 (also, reprint in December, 1956, Producers Monthly).

Rose, Walter, and Wyllie, M. R. J., 1950, Specific surface areas and porosities from microphotographs: Bull. A.A.P.G., v. 34, no. 8, p. 1948.

Winsauer, W. O., Shearin, H. M., Jr., Masson, P. H., and Williams, Milton, 1952, Resistivity of brine-saturated sands in relation to pore geometry: Bull. A.A.P.G., v. 36, no. 2, p. 253.

Wyllie, M. R. J., and Rose, Walter, 1950, Some theoretical considerations related to the quantitative evaluation of the physical characteristics of reservoir rocks from electric logs: Trans. A.I.M.E., v. 189, p. 105.

Illinois State Geological Survey Circular 237
31 p., 11 figs., 5 tables, 1957

CIRCULAR 237

ILLINOIS STATE GEOLOGICAL SURVEY

URBANA